The following example illustrates the design methods presented in ACI 318-05 and IBC 2003. Unless otherwise noted, all referenced table, figure, and equation numbers are from these books. The example presented here is for Two-Way Post-Tensioned Design.

Loads:
Framing Dead Load = selfweight
Superrimposed Dead Load = 25 psf partitions, M/E, misc.
Live Load = 40 psf residential
2 hour fire-rating

Materials:
Concrete: Normal weight 150 pcf
\( f'_{c} = 5,000 \text{ psi} \)
\( f'_{ci} = 3,000 \text{ psi} \)
Rebar: \( f_{y} = 60,000 \text{ psi} \)
PT: Unbonded tendons
\( \frac{1}{2}'' \varphi, 7\)-wire strands, \( A = 0.153 \text{ in}^2 \)
\( f_{pu} = 270 \text{ ksi} \)
Estimated prestress losses = 15 ksi (ACI 18.6)
\( f_{se} = 0.7 (270 \text{ ksi}) - 15 \text{ ksi} = 174 \text{ ksi} \) (ACI 18.5.1)
\( P_{eff} = A \times f_{se} = (0.153)(174 \text{ ksi}) = 26.6 \text{ kips/tendon} \)

Determine Preliminary Slab Thickness
Start with \( L/h = 45 \)
Longest span = 30 ft
\( h = (30 \text{ ft})(12)/45 \)
= 8.0” preliminary slab thickness

Loading
DL = Selfweight = (8in)(150 pcf) = 100 psf
SIDL = 25 psf
LL = 40 psf

IBC 1607.9.1 allows for LL reduction
Exterior bay: \( A_T = (25 \text{ ft})(27 \text{ ft}) = 675 \text{ ft}^2 \)
\( K_{LL} = 1 \)
\( LL = 0.83 LL_o = 33 \text{ psf} \)
Interior bay: \( A_T = (25 \text{ ft})(30 \text{ ft}) = 750 \text{ ft}^2 \)
\( K_{LL} = 1 \)
\( LL = 0.80 LL_o = 32 \text{ psf} \)
DESIGN OF EAST-WEST INTERIOR FRAME
Use Equivalent Frame Method, ACI 13.7 (excluding sections 13.7.7.4-5)
Total bay width between centerlines = 25 ft
Ignore column stiffness in equations for simplicity of hand calculations
No pattern loading required, since LL/DL < 3/4 (ACI 13.7.6)

Calculate Section Properties
Two-way slab must be designed as Class U (ACI 18.3.3), Gross cross-sectional properties allowed
(ACI 18.3.4)
\[
A = bh = (300 \text{ in})(8 \text{ in}) = 2,400 \text{ in}^2
\]
\[
S = bh^2/6 = (300 \text{ in})(8 \text{ in})^2/6 = 3,200 \text{ in}^3
\]

Set Design Parameters
Allowable stresses: Class U (ACI 18.3.3)

At time of jacking (ACI 18.4.1)
\[
f'_{c_{1i}} = 3,000 \text{ psi}
\]
Compression = 0.60 \( f'_{c_{1i}} = 0.6(3,000 \text{ psi}) = 1,800 \text{ psi}
\]
Tension = 3√\( f'_{c_{1i}} = 3\sqrt{3,000} = 164 \text{ psi}
\]

At service loads (ACI 18.4.2(a) and 18.3.3)
\[
f'_{c} = 5,000 \text{ psi}
\]
Compression = 0.45 \( f'_{c} = 0.45(5,000 \text{ psi}) = 2,250 \text{ psi}
\]
Tension = 6√\( f'_{c} = 6\sqrt{5,000} = 424 \text{ psi}
\]

Average precompression limits:
\[
P/A = 125 \text{ psi min. (ACI 18.12.4)}
\]
\[
= 300 \text{ psi max.}
\]

Target load balances:
60%-80% of DL(selfweight) for slabs (good approximation for hand calculation)
For this example: \( 0.75 \times \text{w}_{DL} = 0.75(100 \text{ psf}) = 75 \text{ psf}
\]

Cover Requirements (2-hour fire rating, assume carbonate aggregate)
IBC 2003

Restrained slabs = 3/4" bottom
Unrestrained slabs = 1 1/2" bottom
= 3/4" top
Tendon profile:

Parabolic shape:

For a layout with spans of similar length, the tendons will be typically be located at the highest allowable point at the interior columns, the lowest possible point at the midspans, and the neutral axis at the anchor locations. This provides the maximum drape for load-balancing.

<table>
<thead>
<tr>
<th>Tendon Ordinate</th>
<th>Tendon (CG) Location*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior support - anchor</td>
<td>4.0”</td>
</tr>
<tr>
<td>Interior support - top</td>
<td>7.0”</td>
</tr>
<tr>
<td>Interior span - bottom</td>
<td>1.0”</td>
</tr>
<tr>
<td>End span - bottom</td>
<td>1.75”</td>
</tr>
</tbody>
</table>

(CG) = center of gravity

*Measure from bottom of slab

\[
a_{\text{INT}} = 7.0” - 1.0” = 6.0”
\]

\[
a_{\text{END}} = (4.0” + 7.0”)/2 - 1.75” = 3.75”
\]

eccentricity, e, is the distance from the center to tendon to the neutral axis; varies along the span

Prestress Force Required to Balance 75% of selfweight DL

Since the spans are of similar length, the end span will typically govern the maximum required post-tensioning force. This is due to the significantly reduced tendon drape, \(a_{\text{END}}\).

\[
w_b = 0.75 \cdot w_{DL} = 0.75 \cdot (100 \text{ psf})(25 \text{ ft}) = 1,875 \text{ plf} = 1.875 \text{ k/ft}
\]

Force needed in tendons to counteract the load in the end bay:

\[
P = \frac{w_b L^2}{8a_{\text{end}}}
\]

\[
= \frac{(1.875 \text{ k/ft})(27 \text{ ft})^2}{8(3.75 \text{ in} / 12)}
\]

\[
= 547 \text{ k}
\]
Check Precompression Allowance

Determine number of tendons to achieve 547 k

\[ \# \text{ tendons} = \frac{547 \text{ k}}{26.6 \text{ k/tendon}} \]

= 20.56

Use 20 tendons

Actual force for banded tendons

\[ P_{\text{actual}} = (20 \text{ tendons}) (26.6 \text{ k}) = 532 \text{ k} \]

The balanced load for the end span is slightly adjusted

\[ w_b = \left( \frac{532}{547} \right)(1.875 \text{ k/ft}) = 1.82 \text{ k/ft} \]

Determine actual Precompression stress

\[ \frac{P_{\text{actual}}}{A} = \frac{532 \text{ k}}{(1000) / (2400 \text{ in}^2)} \]

= 221 psi

> 125 psi min.  ok

< 300 psi max.  ok

Check Interior Span Force

\[ P = (1.875 \text{ k/ft})(30 \text{ ft})^2 / [8(6.0 \text{ in / 12})] \]

= 421 k x 532 k

Less force is required in the center bay

For this example, continue the force required for the end spans into the interior span and check the amount of load that will be balanced:

\[ w_b = (532 \text{ k})(8)(6.0 \text{ in / 12}) / (30 \text{ ft})^2 \]

= 2.36 k/ft

\[ w_b/w_{\text{be}} = 94\%; \text{ This value is less than 100\%; acceptable for this design.} \]

East-West interior frame:

Effective prestress force, \( P_{\text{eff}} = 532 \text{ kips} \)
Check Slab Stresses
Separately calculate the maximum positive and negative moments in the frame for the dead, live, and balancing loads. A combination of these values will determine the slab stresses at the time of stressing and at service loads.

Dead Load Moments
\[ w_{DL} = (125 \text{ psf}) \times (25 \text{ ft}) / 1000 = 3.125 \text{ plf} \]

- 3.125 k/ft
  - 240 ft-k
  - 101 ft-k
  - 172 ft-k

Live Load Moments
\[ w_{LL} = (33 \text{ psf}) \times (25 \text{ ft}) / 1000 = 0.825 \text{ plf} \]

- 0.825 k/ft
  - 64 ft-k
  - 27 ft-k
  - 45 ft-k

Total Balancing Moments, \( M_{\text{bal}} \)
\[ w_b = -2.00 \text{ k/ft (average of 3 bays)} \]

- -2.00 k/ft
  - 110 ft-k
  - 65 ft-k
  - 110 ft-k
  - 154 ft-k
  - 154 ft-k
Stage 1: Stresses immediately after jacking (DL + PT) (ACI 18.4.1)

Midspan Stresses

\[ f_{\text{top}} = \frac{-M_{\text{DL}} + M_{\text{bal}}}{S} - \frac{P}{A} \]
\[ f_{\text{bot}} = \frac{+M_{\text{DL}} - M_{\text{bal}}}{S} - \frac{P}{A} \]

Interior Span

\[ f_{\text{top}} = \frac{(-101\text{ft-}k + 65\text{ft-}k)(12)(1000)}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = -135 - 221 = -356 \text{ psi} \] compression < 0.60 f'c = 1800 psi \text{ ok} \]

\[ f_{\text{bot}} = \frac{(101\text{ft-}k - 65\text{ft-}k)(12)(1000)}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = 135 - 221 = -86 \text{ psi} \] compression < 0.60 f'c = 1800 psi \text{ ok} \]

End Span

\[ f_{\text{top}} = \frac{(-172\text{ft-}k + 110\text{ft-}k)(12)(1000)}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = -232 - 221 = -453 \text{ psi} \] compression < 0.60 f'c = 1800 psi \text{ ok} \]

\[ f_{\text{bot}} = \frac{(172\text{ft-}k - 110\text{ft-}k)(12)(1000)}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = 232 - 221 = 11 \text{ psi} \] tension < 3\sqrt{f'c} = 164 psi \text{ ok} \]

Support Stresses

\[ f_{\text{top}} = \frac{+M_{\text{DL}} - M_{\text{bal}}}{S} - \frac{P}{A} \]
\[ f_{\text{bot}} = \frac{-M_{\text{DL}} + M_{\text{bal}}}{S} - \frac{P}{A} \]

\[ f_{\text{top}} = \frac{[(240\text{ft-}k - 154\text{ft-}k)(12)(1000)]}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = 323 - 221 = 102 \text{ psi} \] tension < 3\sqrt{f'c} = 164 psi \text{ ok} \]

\[ f_{\text{bot}} = \frac{[(-240\text{ft-}k + 154\text{ft-}k)(12)(1000)]}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = -323 - 221 = -544 \text{ psi} \] compression < 0.60 f'c = 1800 psi \text{ ok} \]

Stage 2: Stresses at service load (DL + LL + PT) (18.3.3 and 18.4.2)

Midspan Stresses

\[ f_{\text{top}} = \frac{-M_{\text{DL}} - M_{\text{LL}} + M_{\text{bal}}}{S} - \frac{P}{A} \]
\[ f_{\text{bot}} = \frac{+M_{\text{DL}} + M_{\text{LL}} - M_{\text{bal}}}{S} - \frac{P}{A} \]

Interior Span

\[ f_{\text{top}} = \frac{[(-101\text{ft-}k - 27\text{ft-}k+ 65\text{ft-}k)(1000)]}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = -236 - 221 = -457 \text{ psi} \] compression < 0.45 f'c = 2250 psi \text{ ok} \]

\[ f_{\text{bot}} = \frac{[(101\text{ft-}k + 27\text{ft-}k - 65\text{ft-}k)(1000)]}{(3200 \text{ in}^3)} - 221\text{psi} \]
\[ = 236 - 221 = 15 \text{ psi} \] tension < 6\sqrt{f'c} = 424 psi \text{ ok} \]
End Span

\[
f_{\text{top}} = \left[\frac{(-172\text{ft}-k - 45\text{ft}-k + 110\text{ft}-k)(12)(1000)}{3200 \text{ in}^3}\right] - 221\text{psi} \\
= -401 - 221 = -622 \text{ psi compression} < 0.45 f' \ c = 2250 \text{ psi ok}\]

\[
f_{\text{bot}} = \left[\frac{(172\text{ft}-k + 45\text{ft}-k - 110\text{ft}-k)(12)(1000)}{3200 \text{ in}^3}\right] - 221\text{psi} \\
= 401 - 221 = 180 \text{ psi tension} < 6\sqrt{f' \ c} = 424 \text{ psi ok}\]

Support Stresses

\[
f_{\text{top}} = \frac{(+M_{\text{DL}} + M_{\text{LL}} - M_{\text{bal}})}{S} - \frac{P}{A} \\
= \frac{(240\text{ft}-k + 64\text{ft}-k - 154\text{ft}-k)(12)(1000)}{3200 \text{ in}^3} - 221\text{psi} \\
= 563 - 221 = 342 \text{ psi tension} < 6\sqrt{f' \ c} = 424 \text{ psi ok}\]

\[
f_{\text{bot}} = \frac{(-M_{\text{DL}} - M_{\text{LL}} + M_{\text{bal}})}{S} - \frac{P}{A} \\
= \frac{(-240\text{ft}-k - 64 \text{ft}-k + 154\text{ft}-k)(12)(1000)}{3200 \text{ in}^3} - 221\text{psi} \\
= -563 - 221 = -784 \text{ psi compression} < 0.45 f' \ c = 2250 \text{ psi ok}\]

All stresses are within the permissible code limits.

Ultimate Strength

Determine factored moments

The primary post-tensioning moments, \(M_1\), vary along the length of the span.

\[M_1 = P \times e\]

\[e = 0 \text{ in. at the exterior support}\]

\[e = 3.0 \text{ in at the interior support (neutral axis to the center of tendon)}\]

\[M_1 = \frac{(532k)(3.0\text{in})}{(12)} = 133\text{ft-k}\]

The secondary post-tensioning moments, \(M_{\text{sec}}\), vary linearly between supports.

\[M_{\text{sec}} = M_{\text{bal}} - M_1\]

\[= 154 \text{ ft-k} - 133 \text{ ft-k}\]

\[= 21 \text{ ft-k} \text{ at the interior supports}\]

The typical load combination for ultimate strength design is

\[M_u = 1.2 M_{\text{DL}} + 1.6 M_{\text{LL}} + 1.0 M_{\text{sec}}\]

At midspan: \[M_u = 1.2 (172\text{ft-k}) + 1.6 (45\text{ft-k}) + 1.0 (10.5 \text{ ft-k}) = 289 \text{ ft-k}\]

At support: \[M_u = 1.2 (-240\text{ft-k}) + 1.6 (-64\text{ft-k}) + 1.0 (21 \text{ ft-k}) = -370 \text{ ft-k}\]
Determine minimum bonded reinforcement: to see if acceptable for ultimate strength design.

Positive moment region:
- Interior span: \( f_y = 15 \text{ psi} < 2\sqrt{f'c} = 2\sqrt{5,000} = 141 \text{ psi} \)
  - No positive reinforcement required (ACI 18.9.3.1)
- Exterior span: \( f_y = 180 \text{ psi} > 2\sqrt{f'c} = 2\sqrt{5,000} = 141 \text{ psi} \)
  - Minimum positive moment reinforcement required (ACI 18.9.3.2)

\[
\gamma = \frac{f_y}{(f_y + f_c)h} = \frac{[(180)/(180+622)](8 \text{ in})}{1.80 \text{ in}}
\]

\[
N_c = \frac{MDL+LL}{S} * 0.5 * y * l^2
\]

\[
= \frac{[(172 \text{ ft}-k + 45 \text{ ft}-k)(12) / (3,200 \text{ in}^3)](0.5)(1.80 \text{ in})(25 \text{ ft})(12)}{220 \text{ k}}
\]

\[
A_{s, \text{min}} = \frac{N_c}{0.5f_y}
\]

\[
= \frac{(220 \text{ k})}{[0.5(60 \text{ksi})]} = 7.33 \text{ in}^2
\]

Distribute the positive moment reinforcement uniformly across the slab-beam width and as close as practicable to the extreme tension fiber.

\[
A_{s, \text{min}} = (7.33 \text{ in}^2)/(25 \text{ ft}) = 0.293 \text{in}^2/\text{ft}
\]

Use #5 @ 12 in. oc Bottom = 0.31 in²/ft (or equivalent)

Minimum length shall be 1/3 clear span and centered in positive moment region (ACI 18.9.4.1)

Negative moment region:

\[
A_{s, \text{min}} = 0.00075A_{cf} \quad (ACI 18.9.3.3)
\]

Interior supports:

\[
A_{cf} = \text{max. } (8\text{in})[(30\text{ft} + 27\text{ft})/2, 25\text{ft}]^*12
\]

\[
A_{s, \text{min}} = 0.00075(2,736 \text{ in}^2)
\]

\[
= 2.05 \text{ in}^2
\]

\[
= 11 - #4 \text{ Top } (2.20 \text{ in}^2)
\]

Exterior supports:

\[
A_{cf} = \text{max. } (8\text{in})[(27\text{ft}/2), 25\text{ft}]^*12
\]

\[
A_{s, \text{min}} = 0.00075(2,400 \text{ in}^2)
\]

\[
= 1.80 \text{ in}^2
\]

\[
= 9 - #4 \text{ Top } (1.80 \text{ in}^2)
\]

Must span a minimum of 1/6 the clear span on each side of support (ACI 18.9.4.2)

At least 4 bars required in each direction (ACI 18.9.3.3)

Place top bars within 1.5h away from the face of the support on each side (ACI 18.9.3.3)

\[
= 1.5 \text{ (8 in)}
\]

\[
= 12 \text{ in}
\]

Maximum bar spacing is 12” (ACI 18.9.3.3)
Check minimum reinforcement if it is sufficient for ultimate strength

\[ M_n = \left( A_{sfy} + A_{ps}f_{ps} \right) \left( d-a/2 \right) \]

\( d \) = effective depth

\( A_{ps} = 0.153 \text{in}^2 \times \text{(number of tendons)} \)

\( = 0.153 \text{in}^2 \times 20 \text{ tendons} \)

\( = 3.06 \text{ in}^2 \)

\( f_{ps} = f_{se} + 10,000 + \frac{(f'_{cbd})(25ft*12)}{300A_{ps}} \) for slabs with \( L/h > 35 \) (ACI 18.7.2)

\( = 174,000 \text{psi} + 10,000 + \frac{(5,000 \text{psi})(25 \text{ft} \times 12)}{300(3.06 \text{ in}^2)} \)

\( = 184,000 \text{psi} + 1634d \)

\( a = \frac{(A_{sfy} + A_{ps}f_{ps})}{0.85f'_{cb}} \)

At supports

\( d = 8\" - 3/4\" - 1/4\" = 7\" \)

\( f_{ps} = 184,000 \text{psi} + 1634(7\") = 195,438 \text{psi} \)

\( a = \frac{((2.20 \text{ in}^2)(60 \text{ ksi}) + (3.06 \text{ in}^2)(195 \text{ kksi})}{((0.85)(5 \text{ kksi})(25 \text{ft} \times 12))} = 0.57 \)

\( q_{M_n} = 0.9 \left[ (2.20 \text{ in}^2)(60 \text{ ksi}) + (3.06 \text{ in}^2)(195 \text{ kksi}) \right] \left[ 7\" - (0.57)/2 \right]/12 \)

\( = 0.9(728k)(6.72\text{in})/12 \)

\( = 367 \text{ ft-k} < 370 \text{ ft-k} \) Reinforcement for ultimate strength requirements governs

\[ A_{s, \text{reqd}} = 2.30\text{in} \]

**12 - #4 Top at interior supports**

**9 - #4 Top at exterior supports**

When reinforcement is provided to meet ultimate strength requirements, the minimum lengths must also conform to the provision of ACI 318-05 Chapter 12. (ACI 18.9.4.3)

At midspan (end span)

\( d = 8\" - 1\frac{1}{2}\" - 1/4\" = 6\frac{1}{4}\" \)

\( f_{ps} = 184,000 \text{psi} + 1634(6.25\") = 194,212 \text{psi} \)

\( a = \frac{((7.33 \text{ in}^2)(60 \text{ ksi}) + (3.06 \text{ in}^2)(194 \text{ kksi})}{((0.85)(5 \text{ kksi})(25 \text{ft} \times 12))} = 0.81 \)

\( q_{M_n} = 0.9 \left[ (7.33 \text{ in}^2)(60 \text{ ksi}) + (3.06 \text{ in}^2)(194 \text{ kksi}) \right] [6.25\" - (0.81)/2]/12 \)

\( = 0.9(1033k)(5.85\text{in})/12 \)

\( = 453 \text{ ft-k} > 289 \text{ ft-k} \) Minimum reinforcement ok

**#5 @ 12\" oc Bottom at end spans**

This is a simplified hand calculation for a post-tensioned two-way plate design. A detailed example can be found in the PCA Notes on ACI 318-05 Building Code Requirements for Structural Concrete.