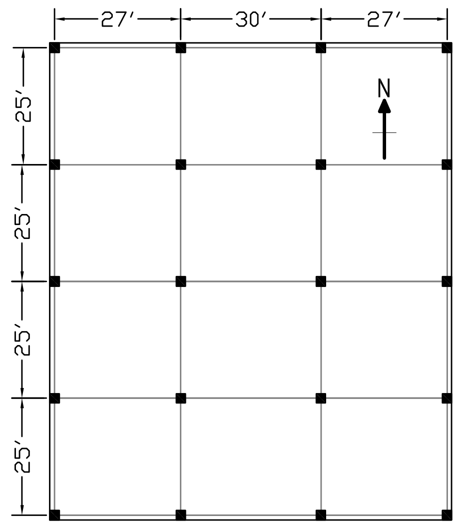
Two-Way Post-Tensioned Design

The following example illustrates the design methods presented in ACI 318-05 and IBC 2003. Unless otherwise noted, all referenced table, figure, and equation numbers are from these books. The example presented here is for Two-Way Post-Tensioned Design.

Loads:

Framing Dead Load = selfweight
Superrimposed Dead Load = 25 psf partitions, M/E, misc.
Live Load = 40 psf residential
2 hour fire-rating



Materials:

Concrete: Normal weight 150 pcf

f'_c = 5,000 psi

f'_{ci} = 3,000 psi

Rebar: f_y = 60,000 psi

PT: Unbonded tendons

$^1/_2$"φ, 7-wire strands, A = 0.153 in^2

f_{pu} = 270 ksi

Estimated prestress losses = 15 ksi (ACI 18.6)

f_{se} = 0.7 (270 ksi) - 15 ksi = 174 ksi (ACI 18.5.1)

$P_{eff} = A*f_{se}$ = (0.153)(174 ksi) = 26.6 kips/tendon

Determine Preliminary Slab Thickness

Start with L/h = 45
Longest span = 30 ft

h = (30 ft)(12)/45

= 8.0" preliminary slab thickness

Loading

DL = Selfweight = (8in)(150 pcf) = 100 psf

SIDL = 25 psf

LL_o = 40 psf

IBC 1607.9.1 allows for LL reduction

Exterior bay: A_T = (25 ft)(27 ft) = 675 ft^2

K_{LL} = 1

LL = 0.83 LL_o = 33 psf

Interior bay: A_T = (25 ft)(30 ft) = 750 ft^2

K_{LL} = 1

LL = 0.80 LL_o = 32 psf

DESIGN OF EAST-WEST INTERIOR FRAME

Use Equivalent Frame Method, ACI 13.7 (excluding sections 13.7.7.4-5)
Total bay width between centerlines = 25 ft
Ignore column stiffness in equations for simplicity of hand calculations
No pattern loading required, since LL/DL < 3/4 (ACI 13.7.6)

Calculate Section Properties

Two-way slab must be designed as Class U (ACI 18.3.3), Gross cross-sectional properties allowed (ACI 18.3.4)

$A = bh = (300\text{ in})(8\text{ in}) = 2{,}400\text{ in}^2$

$S = bh^2/6 = (300\text{ in})(8\text{ in})^2/6 = 3{,}200\text{ in}^3$

Set Design Parameters

Allowable stresses: Class U (ACI 18.3.3)

At time of jacking (ACI 18.4.1)

f'_{ci} = 3,000 psi

Compression = $0.60\, f'_{ci} = 0.6(3{,}000\text{ psi}) = 1{,}800$ psi

Tension = $3\sqrt{f'_{ci}} = 3\sqrt{3{,}000} = 164$ psi

At service loads (ACI 18.4.2(a) and 18.3.3)

f'_c = 5,000 psi

Compression = $0.45\, f'_c = 0.45(5{,}000\text{ psi}) = 2{,}250$ psi

Tension = $6\sqrt{f'_c} = 6\sqrt{5{,}000} = 424$ psi

Average precompression limits:
P/A = 125 psi min. (ACI 18.12.4)
= 300 psi max.

Target load balances:

60%-80% of DL(selfweight) for slabs (good approximation for hand calculation)

For this example: $0.75\, w_{DL} = 0.75(100\text{ psf}) = 75$ psf

Cover Requirements (2-hour fire rating, assume carbonate aggregate)
IBC 2003

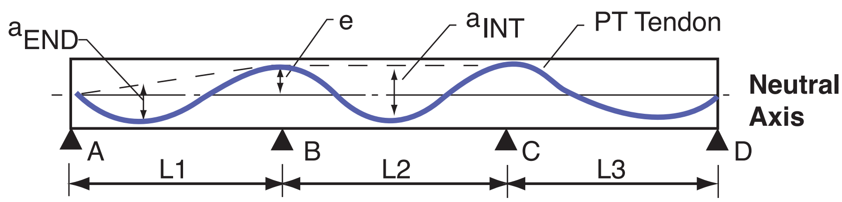
Restrained slabs = 3/4″ bottom

Unrestrained slabs = 1 1/2″ bottom
= 3/4″ top

Tendon profile:

Parabolic shape;

For a layout with spans of similar length, the tendons will be typically be located at the highest allowable point at the interior columns, the lowest possible point at the midspans, and the neutral axis at the anchor locations. This provides the maximum drape for load-balancing.



Continuous Post-Tensioned Beam

Tendon Ordinate	Tendon (CG) Location*
Exterior support - anchor	4.0"
Interior support - top	7.0"
Interior span - bottom	1.0"
End span - bottom	1.75"

(CG) = center of gravity

*Measure from bottom of slab

a_{INT} = 7.0" - 1.0" = 6.0"

a_{END} = (4.0" + 7.0")/2 - 1.75" = 3.75"

eccentricity, e, is the distance from the center to tendon to the neutral axis; varies along the span

Prestress Force Required to Balance 75% of selfweight DL

Since the spans are of similar length, the end span will typically govern the maximum required post-tensioning force. This is due to the significantly reduced tendon drape, a_{END}.

$w_b = 0.75\ w_{DL}$ = 0.75 (100 psf)(25 ft) = 1,875 plf = 1.875 k/ft

Force needed in tendons to counteract the load in the end bay:

$P = w_b L^2 / 8a_{end}$

= (1.875 k/ft)(27 ft)^2 / [8(3.75 in / 12)]

= 547 k

Check Precompression Allowance

Determine number of tendons to achieve 547 k

tendons = (547 k) / (26.6 k/tendon)

= 20.56

Use 20 tendons

Actual force for banded tendons

P_{actual} = (20 tendons) (26.6 k) = 532 k

The balanced load for the end span is slightly adjusted

w_b = (532/547)(1.875 k/ft) = 1.82 k/ft

Determine actual Precompression stress

P_{actual} /A = (532 k)(1000) / (2,400 in^2)

= 221 psi > 125 psi min. ok

< 300 psi max. ok

Check Interior Span Force

P = (1.875 k/ft)(30 ft)^2 / [8(6.0 in / 12)]

= 421 k < 532 k

Less force is required in the center bay

For this example, continue the force required for the end spans into the interior span and check the amount of load that will be balanced:

w_b = (532 k)(8)(6.0 in /12) / (30 ft)^2

= 2.36 k/ft

w_b/w_{DL} = 94%; This value is less than 100%; acceptable for this design.

East-West interior frame:

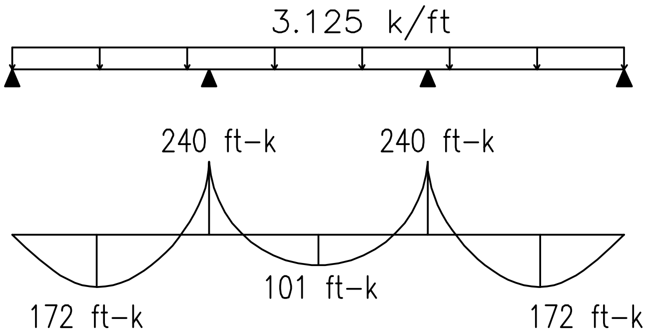
Effective prestress force, P_{eff} = 532 kips

Check Slab Stresses

Separately calculate the maximum positive and negative moments in the frame for the dead, live, and balancing loads. A combination of these values will determine the slab stresses at the time of stressing and at service loads.

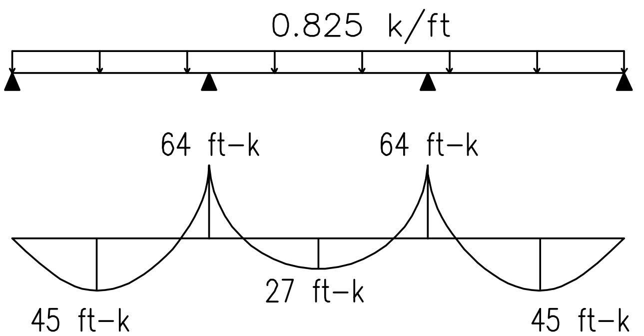
Dead Load Moments

w_{DL} = (125 psf) (25 ft) / 1000 = 3.125 plf



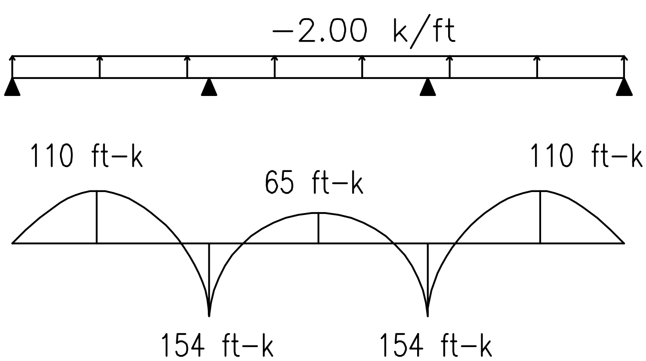
Live Load Moments

w_{LL} = (33 psf) (25 ft) / 1000 = 0.825 plf



Total Balancing Moments, M_{bal}

w_b = -2.00 k/ft (average of 3 bays)



Two-Way Post-Tensioned Design

Stage 1: Stresses immediately after jacking (DL + PT) (ACI 18.4.1)

Midspan Stresses

$f_{top} = (-M_{DL} + M_{bal})/S - P/A$

$f_{bot} = (+M_{DL} - M_{bal})/S - P/A$

Interior Span

$f_{top} = [(-101ft\text{-}k + 65ft\text{-}k)(12)(1000)]/(3200\ in3) - 221psi$

$= -135 - 221 = -356$ psi compression < 0.60 f'ci = 1800 psi ok

$f_{bot} = [(101ft\text{-}k - 65ft\text{-}k)(12)(1000)]/(3200\ in3) - 221psi$

$= 135 - 221 = -86$ psi compression < 0.60 f'ci = 1800 psi ok

End Span

$f_{top} = [(-172ft\text{-}k + 110ft\text{-}k)(12)(1000)]/(3200\ in^3) - 221psi$

$= -232 - 221 = -453$ psi compression < $0.60\ f'_{ci}$ = 1800 psi ok

$f_{bot} = [(172ft\text{-}k - 110ft\text{-}k)(12)(1000)]/(3200\ in^3) - 221psi$

$= 232 - 221 = 11$ psi tension < $3\sqrt{f'_{ci}}$ = 164 psi ok

Support Stresses

$f_{top} = (+M_{DL} - M_{bal})/S - P/A$

$f_{bot} = (-M_{DL} + M_{bal})/S - P/A$

$f_{top} = [(240ft\text{-}k - 154ft\text{-}k)(12)(1000)]/(3200\ in^3) - 221psi$

$= 323 - 221 = 102$ psi tension < $3\sqrt{f'_{ci}}$ = 164 psi ok

$f_{bot} = [(-240ft\text{-}k + 154ft\text{-}k)(12)(1000)]/(3200\ in^3) - 221psi$

$= -323 - 221 = -544$ psi compression < $0.60\ f'_{ci}$ = 1800 psi ok

Stage 2: Stresses at service load (DL + LL + PT) (18.3.3 and 18.4.2)

Midspan Stresses

$f_{top} = (-M_{DL} - M_{LL} + M_{bal})/S - P/A$

$f_{bot} = (+M_{DL} + M_{LL} - M_{bal})/S - P/A$

Interior Span

$f_{top} = [(-101ft\text{-}k - 27ft\text{-}k + 65ft\text{-}k)(1000)]/(3200\ in^3) - 221psi$

$= -236 - 221 = -457$ psi compression < $0.45\ f'_c$ = 2250 psi ok

$f_{bot} = [(101ft\text{-}k + 27ft\text{-}k - 65ft\text{-}k)(1000)]/(3200\ in3) - 221psi$

$= 236 - 221 = 15$ psi tension < $6\sqrt{f'_c}$ = 424 psi ok

End Span

f_{top} = [(-172ft-k - 45ft-k + 110ft-k)(12)(1000)]/(3200 in^3) - 221psi

= -401 - 221 = -622 psi compression $< 0.45 f'_c$ = 2250 psi ok

f_{bot} = [(172ft-k + 45ft-k - 110ft-k)(12)(1000)]/(3200 in^3) - 221psi

= 401 - 221 = 180 psi tension $< 6\sqrt{f'_c}$ = 424 psi ok

Support Stresses

$f_{top} = (+M_{DL} + M_{LL} - M_{bal})/S - P/A$

$f_{bot} = (-M_{DL} - M_{LL} + M_{bal})/S - P/A$

f_{top} = [(240ft-k + 64ft-k - 154ft-k)(12)(1000)]/(3200 in^3) - 221psi

= 563 - 221 = 342 psi tension $< 6\sqrt{f'_c}$ = 424 psi ok

f_{bot} = [(-240ft-k - 64 ft-k + 154ft-k)(12)(1000)]/(3200 in^3) - 221psi

= -563 - 221 = -784 psi compression $< 0.45 f'_c$ = 2250 psi ok

All stresses are within the permissible code limits.

Ultimate Strength

Determine factored moments
The primary post-tensioning moments, M_1, vary along the length of the span.

$M_1 = P * e$

e = 0 in. at the exterior support

e = 3.0 in at the interior support (neutral axis to the center of tendon)

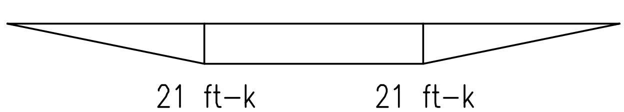
M_1 = (532k)(3.0in) / (12) = 133ft-k

The secondary post-tensioning moments, M_{sec}, vary linearly between supports.

$M_{sec} = M_{bal} - M_1$

= 154 ft-k - 133 ft-k

= 21 ft-k   at the interior supports



The typical load combination for ultimate strength design is

$M_u = 1.2\ M_{DL} + 1.6\ M_{LL} + 1.0\ M_{sec}$

At midspan: M_u = 1.2 (172ft-k) + 1.6 (45ft-k) + 1.0 (10.5 ft-k) = 289 ft-k

At support: M_u = 1.2 (-240ft-k) + 1.6 (-64ft-k) + 1.0 (21 ft-k) = -370 ft-k

Determine minimum bonded reinforcement: to see if acceptable for ultimate strength design.

Positive moment region:

Interior span: f_t = 15 psi < $2\sqrt{f'_c} = 2\sqrt{5{,}000}$ = 141 psi

No positive reinforcement required (ACI 18.9.3.1)

Exterior span: f_t = 180 psi > $2\sqrt{f'_c} = 2\sqrt{5{,}000}$ = 141 psi

Minimum positive moment reinforcement required (ACI 18.9.3.2)

$y = f_t/(f_t + f_c)h$

$= [(180)/(180+622)](8\text{ in})$

$= 1.80\text{ in}$

$N_c = M_{DL+LL}/S * 0.5 * y * \ell_2$

$= [(172\text{ ft-k} + 45\text{ ft-k})(12) / (3{,}200\text{ in}^3)](0.5)(1.80\text{ in})(25\text{ft})(12)$

$= 220\text{ k}$

$A_{s,\,min} = N_c / 0.5_{fy}$

$= (220\text{ k}) / [0.5(60\text{ksi})]$

$= 7.33\text{ in}^2$

Distribute the positive moment reinforcement uniformly across the slab-beam width and as close as practicable to the extreme tension fiber.

$A_{s,\,min} = (7.33\text{ in}^2)/(25\text{ ft})$

$= 0.293\text{in}^2/\text{ft}$

Use #5 @ 12 in. oc Bottom = 0.31 in^2/ft (or equivalent)

Minimum length shall be 1/3 clear span and centered in positive moment region (ACI 18.9.4.1)

Negative moment region:

$A_{s,\,min} = 0.00075A_{cf}$ (ACI 18.9.3.3)

Interior supports:

A_{cf} = max. (8in)[(30ft + 27ft)/2, 25ft]*12

$A_{s,\,min} = 0.00075(2{,}736\text{ in}^2)$

$= 2.05\text{ in}^2$

= 11 - #4 Top (2.20 in^2)

Exterior supports:

A_{cf} = max. (8in)[(27ft/2), 25ft]*12

$A_{s,\,min} = 0.00075(2{,}400\text{ in}^2)$

$= 1.80\text{ in}^2$

= 9 - #4 Top (1.80 in^2)

Must span a minimum of 1/6 the clear span on each side of support (ACI 18.9.4.2)

At least 4 bars required in each direction (ACI 18.9.3.3)

Place top bars within 1.5h away from the face of the support on each side (ACI 18.9.3.3)

= 1.5 (8 in)

= 12 in

Maximum bar spacing is 12" (ACI 18.9.3.3)

Check minimum reinforcement if it is sufficient for ultimate strength

M_n = $(A_s f_y + A_{ps} f_{ps})$ (d-a/2)

d = effective depth

A_{ps} = 0.153in^2*(number of tendons)

= 0.153in^2*(20 tendons)

= 3.06 in^2

f_{ps} = f_{se} + 10,000 + $(f'_c bd)/(300A_{ps})$ for slabs with L/h > 35 (ACI 18.7.2)

= 174,000psi + 10,000 + [(5,000psi)(25ft*12)d]/[(300)(3.06 in^2)]

= 184,000psi + 1634d

a = $(A_s f_y + A_{ps} f_{ps}) / (0.85 f'_c b)$

At supports

d = 8" - 3/4 " - 1/4" = 7"

f_{ps} = 184,000psi + 1634(7") = 195,438psi

a = [(2.20 in^2)(60 ksi) + (3.06 in^2)(195ksi)]/[(0.85)(5ksi)(25ft*12)] = 0.57

φM_n = 0.9 [(2.20 in^2)(60 ksi) + (3.06 in^2)(195ksi)][7" - (0.57)/2]/12

= 0.9 (728k)(6.72in)/12

= 367 ft-k < 370ft-k Reinforcement for ultimate strength requirements governs

$A_{s,\,reqd}$ = 2.30in

12 - #4 Top at interior supports
9 - #4 Top at exterior supports

When reinforcement is provided to meet ultimate strength requirements, the minimum lengths must also conform to the provision of ACI 318-05 Chapter 12. (ACI 18.9.4.3)

At midspan (end span)

d = 8" - 1 1/2" - 1/4" = 6 1/4"

f_{ps} = 184,000psi + 1634(6.25") = 194,212psi

a = [(7.33 in2)(60 ksi) + (3.06 in^2)(194ksi)]/[(0.85)(5ksi)(25ft*12)] = 0.81

φM_n = 0.9 [(7.33 in^2)(60 ksi) + (3.06 in^2)(194ksi)][6.25" - (0.81)/2]/12

= 0.9 (1033k)(5.85in)/12

= 453 ft-k > 289 ft-k Minimum reinforcement ok

#5 @ 12" oc Bottom at end spans

This is a simplified hand calculation for a post-tensioned two-way plate design. A detailed example can be found in the PCA *Notes on ACI 318-05 Building Code Requirements for Structural Concrete.*